OPTICAL METHODS FOR THE MEASUREMENT OF COMPLEX DIELECTRIC AND MAGNETIC CONSTANTS AT CENTIMETER AND MILLIMETER WAVELENGTHS*

T. E. Talpey
Bell Telephone Laboratories, Inc.
Murray Hill, New Jersey

ABSTRACT

A method is described which permits the determination of the complex dielectric constant, $\varepsilon^* = \varepsilon_o k_e (1-j \tan \delta_e)$, and the complex permeability, $\mu^* = \mu_o k_m (1-j \tan \delta_m)$, using free space transmission and reflection from a plane sheet of the sample dielectric. The procedure represents an extension of that used at optical frequencies. Differences arise however, due to the fact that the assumptions of $k_m = 1$ and $\tan \delta_m = 0$, which are made in the optical theory, are not always valid at millimeter wavelengths.

A measurement of the Brewster angle, or the angle of incidence which results in minimum reflection of a parallel-polarized wave, together with a measurement of the reflection coefficient for a perpendicularly polarized wave incident at the same angle, will permit the determination of the two quantities k_e and k_m. Measurement of the transmission coefficient with incidence at the Brewster angle, together with a measurement of the ratio of the reflection coefficients with parallel and perpendicular polarizations (at the Brewster angle), will allow $\tan \delta_e$ and $\tan \delta_m$ to be determined.

A method of handling the experimental data to correct for multiple reflections within the sample is presented, and the effect of losses on the determination of k_e and k_m is discussed. These considerations indicate that the method in its present form can be applied only to those materials for which $\tan(\delta_e + \delta_m)$ is less than 0.1.

The experimental equipment described includes a Klystron for generating eight millimeter waves, a coaxial wavemeter which permits determination of the wavelength to 0.01 millimeter, a calibrated cut-off attenuator, and a special crystal detector. The mechanical arrangement permits adjustment and accurate measurement of the angle of incidence, as well as change of polarization by rotation of the horn antennas.

A discussion of experimental results concludes the paper.

INTRODUCTION

The electric and magnetic properties of a substance can be specified by the following two quantities:

(1) Complex dielectric constant =
$$\varepsilon^* = \varepsilon' - j\varepsilon'' = \varepsilon_o k_e (1-j \tan \delta_e)$$

(2) Complex Permeability =
$$\mu^* = \mu' - j\mu'' = \mu_o k_m (1-j \tan \delta_m).$$

In these equations ε_o represents the dielectric constant of free space, equal to $\frac{10^{-9}}{36\pi}$ farads per meter, μ_o represents the permeability of free space and is equal to $4\pi \times 10^{-7}$ henries per meter, k_e is the relative dielectric constant, k_m is the relative permeability, δ_e is the dielectric loss angle, and δ_m is the magnetic loss angle.

Among the possibilities for measuring dielectric constants and losses of materials at very high frequencies it is of interest to consider a process employing free space transmission and reflection, as indicated by Fig. 1.

If a dielectric sheet is placed in a plane-polarized electromagnetic field in such a manner that the electric field is parallel to the plane of incidence, an angle of incidence can be found for which the reflected field is a minimum. This angle, θ_B, is called the Brewster or pseudo-Brewster angle, after its counterpart in optical terminology.

It can be shown that for the case of small losses and relative permeability equal to unity the relative dielectric constant, k_e, is given by

$$(3) \qquad \tan^2 \theta_B = k_e .$$

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By determining the angle which gives the minimum reflected field it is possible to calculate the dielectric constant of the reflecting material.

When the sheet of material is placed in the electromagnetic field with the angle of incidence equal to the Brewster angle, measurement of the ratio, $\frac{1}{r^2}$, of the power transmitted through the sheet to the power transmitted without the sheet in place permits the determination of the loss angle, by means of the following relationship[1]:

$$(4) \quad \tan \delta_e = \frac{\lambda}{\pi d} \frac{\ln r}{\sqrt{k_e + 1}} + O(\tan^2 \delta_e)$$

where λ = free space wavelength
d = thickness of the sample.

$O(\tan^2 \delta_e)$ represents all terms of second order and higher in $\tan \delta_e$. If the losses are small, the second order terms in $\tan \delta_e$ are negligible and the value of $\tan \delta_e$ is given by Eq. (4).

The application of equations (3) and (4) to the measurement of dielectric constants and losses is not practical at frequencies of the order of several megacycles. However, at millimeter wavelengths the required dimensions of the sheet (area sufficient to avoid diffraction effects at the edges of the sample and thickness sufficient to give a reasonable value of "r" in Eq. (4)) become easily realizable for a large number of dielectrics. For these frequencies the method becomes a practical method for the measurement of dielectric properties. Moreover it provides a substitute for methods based on propagation in waveguides, which become difficult when the wavelength is too small.

This paper presents a modification of equations (3) and (4) which holds when k_m is not equal to unity and $\tan \delta_m$ is not zero. A description is given of the additional measurements and calculations necessary for finding the four quantities k_e, k_m, $\tan \delta_e$, and $\tan \delta_m$ when the losses are small. A description is also given of the apparatus constructed and used for the measurement of dielectric and magnetic constants at a wavelength of eight millimeters. Finally the results of measurements on several materials are given.

Fundamental Formulas Relative to Reflection and Transmission

It is useful to recall first of all several fundamental relations which will be needed.

Consider a plane-polarized electromagnetic wave incident on a dielectric sheet in air. Two general cases are possible:

(I) the plane of polarization is parallel to the plane of incidence,

(II) the plane of polarization is perpendicular to the plane of incidence.

For the first case, it can be shown that the reflected electric field and the transmitted field are given respectively by(2):

$$(5) \quad E_{r\parallel} = E_i \frac{\frac{\varepsilon}{\varepsilon_o} \cos \theta - \sqrt{\frac{\mu}{\mu_o}\frac{\varepsilon}{\varepsilon_o} - \sin^2 \theta}}{\frac{\varepsilon}{\varepsilon_o} \cos \theta + \sqrt{\frac{\mu}{\mu_o}\frac{\varepsilon}{\varepsilon_o} - \sin^2 \theta}}$$

$$(6) \quad E_{t\parallel} = E_i \frac{2\sqrt{\frac{\mu}{\mu_o}\frac{\varepsilon}{\varepsilon_o}} \cos \theta}{\frac{\varepsilon}{\varepsilon_o} \cos \theta + \sqrt{\frac{\mu}{\mu_o}\frac{\varepsilon}{\varepsilon_o} - \sin^2 \theta}} .$$

The subscript "$\parallel$" signifies that these equations hold for the case of polarization parallel to the plane of incidence. E_i is the incident electric field strength.

For the second case the two magnetic fields (reflected and transmitted) are given by:

$$(7) \quad -H_{r\perp} = H_i \frac{\frac{\mu}{\mu_o} \cos \theta - \sqrt{\frac{\mu}{\mu_o}\frac{\varepsilon}{\varepsilon_o} - \sin^2 \theta}}{\frac{\mu}{\mu_o} \cos \theta + \sqrt{\frac{\mu}{\mu_o}\frac{\varepsilon}{\varepsilon_o} - \sin^2 \theta}}$$

$$(8) \quad H_{t\perp} = H_i \frac{2\sqrt{\frac{\mu}{\mu_o}\frac{\varepsilon}{\varepsilon_o}} \cos \theta}{\frac{\mu}{\mu_o} \cos \theta + \sqrt{\frac{\mu}{\mu_o}\frac{\varepsilon}{\varepsilon_o} - \sin^2 \theta}} .$$

As functions of the electric field (after substitution of the relation $E = \sqrt{\frac{\mu}{\varepsilon}}\, H$):

$$(9)\quad -E_{r\perp} = E_i \frac{\frac{\mu}{\mu_o}\cos\theta - \sqrt{\frac{\mu\,\varepsilon}{\mu_o\,\varepsilon_o} - \sin^2\theta}}{\frac{\mu}{\mu_o}\cos\theta + \sqrt{\frac{\mu\,\varepsilon}{\mu_o\,\varepsilon_o} - \sin^2\theta}}$$

$$(10)\quad E_{t\perp} = E_i \frac{2\frac{\mu}{\mu_o}\cos\theta}{\frac{\mu}{\mu_o}\cos\theta + \sqrt{\frac{\mu\,\varepsilon}{\mu_o\,\varepsilon_o} - \sin^2\theta}}.$$

The subscript "$\perp$" signifies that these equations hold for the case of perpendicular polarization. These relations, as well as the relations for the case of parallel polarization, hold even for complex values of ε and μ.

The Case of Zero Losses

The calculation of Brewster's angle for a dielectric without losses will be presented first. Later the error caused by finite losses will be discussed. Substituting (1) and (2) into (5) yields, for a parallel-polarized wave, zero losses:

$$(11)\quad E_{r\parallel} = E_i \frac{k_e\cos\theta - \sqrt{k_e k_m - \sin^2\theta}}{k_e\cos\theta + \sqrt{k_e k_m - \sin^2\theta}}.$$

The reflected field becomes zero when the numerator is zero and the particular angle which satisfies this condition is called the Brewster angle, θ_B. For zero reflected field then:

$$(12)\quad \tan^2\theta_B = \frac{k_e(k_e - k_m)}{k_e k_m - 1} \text{ when } k_e > k_m .$$

For most dielectrics k_m is equal to unity and Eq. (12) reduces to Eq. (3). It should be noted that Eq. (12) holds only for the case where k_e is larger than k_m; when k_e is less than k_m Eq. (12) has no real solution and there is no minimum in the reflected field. However, examination of Eq. (9) for perpendicular polarization, reveals that $E_\perp$ does have a minimum (equal to zero losses) for an angle of incidence satisfying the condition:

$$(13)\quad \tan^2\theta_B = \frac{k_m(k_m - k_e)}{k_m k_e - 1} \quad \text{when } k_m > k_e.$$

It is apparent that there is a similarity in form between the equations for parallel polarization and those for perpendicular polarization(3). It turns out that all the relations for perpendicular polarization can be obtained simply by replacing ε by μ and vice versa in the corresponding relations for parallel polarization. Hence it will not be necessary to treat separately the case of $k_m > k_e$; it is sufficient to note here that for dielectrics in which $k_m > k_e$ the formulas derived in this paper can be used by replacing k_e by k_m, $\tan\delta_e$ by $\tan\delta_m$, etc. At the same time, measurements of the electric field parallel to the plane of incidence must be replaced by corresponding measurements of the field perpendicular to the plane of incidence and vice versa.

Eq. (12) above, does not permit the calculation of k_e unless the value of k_m is known. A second relation between k_e and k_m is obtained when the reflected field for the case of perpendicular polarization is considered. For zero losses, Eq. (9) yields

$$(14)\quad \frac{E_{r\perp}}{E_i} = -\frac{k_m\cos\theta - \sqrt{k_e k_m - \sin^2\theta}}{k_m\cos\theta + \sqrt{k_e k_m - \sin^2\theta}}.$$

If the angle of incidence is equal to Brewster's angle, already determined by an experiment with parallel polarization, then:

$$(15)\quad \left[\frac{E_{r\perp}}{E_i}\right]_{\theta=\theta_B} = \frac{1}{u} = \frac{k_e - k_m}{k_e + k_m}.$$

In order to obtain the value of the incident field, the dielectric sample is replaced by a metallic sheet or alternatively the antennas (transmitting and receiving) are aligned without changing the distance traveled by the electromagnetic wave. When it is known in advance that k_m is equal to unity, the measurement of the reflection coefficient is not necessary.

Effects of Parasitic Reflections

Reflections at the back of the dielectric sample and multiple reflections inside the sample have been neglected in deriving Eq. (15). These effects are treated below where it is shown that a series of successive approximations can be used to correct for the errors introduced by multiple reflections.

Reflections from the transmitting and receiving horns are the cause of another error. The receiving horn reflects part of the incident energy and sends it back toward the transmitting horn where it is again partially reflected. This process is repeated indefinitely. The result is a stationary wave between the horns, the deflection of the detector varying from minimum to maximum as the receiving horn is moved from a minimum to a maximum of the interference pattern. Analysis of this effect shows that the error is effectively annulled by using the average of two readings of the detector, the second being taken after a quarter-wave displacement of the receiving horn with the transmitted power held constant. The compensation is improved by inserting an attenuating sheet between the two horns. The sheet is placed in such a manner that the high-frequency wave strikes it at its own Brewster angle, thereby avoiding additional reflections due to the insertion of this attenuating sheet.

Effect of Losses on the Relations Giving k_e and k_m

The effect of losses on relations at the Brewster angle remains to be examined. When losses are present the angle giving minimum reflection is not precisely equal to the angle which satisfies Eq. (12). Unfortunately the problem of finding the exact value of θ_B is too complex to permit an analytic investigation without making a large number of simplifying approximations.

Numerical calculations which were made with the help of a calculating machine for several typical cases show that for most of the range of measured values the variation of θ_B due to losses can be neglected. However, when $\tan \delta_e$ or $\tan(\delta_e+\delta_m)$ is greater than 0.1 it is necessary to take the losses into account and correct the measured value of θ_B .

Measurement of the Sum of the Losses

The sum of the dielectric and magnetic loss angles is obtained by a transmission measurement, the parallel-polarized wave traversing the sample at Brewster's angle. The amplitude of the wave is modified not only by the coefficients of transmission at the faces but also by attenuation inside the sample.

Consider a wave incident at an angle θ to the normal at the surface of a dielectric as indicated by Fig. 2. If the wave incident at point A is represented by $E_o e^{j\omega t}$, the wave emerging at point B is given by:

$$(16) \qquad E_i = T_1 \cdot T_2 \cdot E_o e^{j\omega t} \cdot e^{\cdot jk\ell}$$

where $K = \omega\sqrt{\mu\varepsilon}$ = propagation constant of the wave in the dielectric

$\ell = \dfrac{d}{\cos \psi}$ = length of the path traversed by the wave in the dielectric.

It should be noted that the propagation constant K is complex and that the angle ψ in Fig. 2 is not the angle φ given by Snell's law*. A complex angle φ results when Snell's law is applied to a dielectric with losses. It is necessary to know the real angle, ψ, which Poynting's vector makes with the normal to the surface of the dielectric. The calculation of this angle has been made[4] and a convenient way to express the result is:

$$(17) \qquad \cos \psi = \frac{p}{\sqrt{p^2 + \sin^2 \theta}}$$

where "p" is given by the relations

$$(18) \qquad 2p^2 = \sqrt{(a^2-\sin^2\theta)^2 + a^2\tan^2(\delta_e + \delta_m)} + (a^2 - \sin^2 \theta)$$

$$(19) \qquad a^2 = k_e k_m(1 - \tan \delta_e \tan \delta_m) .$$

In order to calculate the attenuation inside the dielectric sample it is convenient to deal with the logarithm of the absolute value of Eq. (16). Calculation of the constant K by using Eqs. (1) and (2) and substitution of Eq. (18) in Eq. (16), yields

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* $\sin \theta = \sqrt{\dfrac{\mu\varepsilon}{\mu_o\varepsilon_o}} \sin \varphi$. This equation is valid even when the quantities μ and ε are complex.

$$(20)\quad \ln\left|\frac{E_i}{E_o}\right| = |T_1 \cdot T_2| - \frac{2\pi d}{\lambda}\sqrt{k_e k_m}\sqrt{\frac{\tan\delta_e + \tan\delta_m}{2\tan\frac{\delta_e+\delta_m}{2}}}\;\frac{\sqrt{p^2+\sin^2\theta}}{2}\;\tan\frac{\delta_e+\delta_m}{2}\,.$$

Eq. (20) is exact; no approximation has been made in obtaining it. Assuming that the losses are small so that

$$\sqrt{\frac{\tan\delta_e + \tan\delta_m}{2\tan\frac{\delta_e+\delta_m}{2}}} \cong 1.0,$$ and setting $\theta=\theta_B$ so that* $|T_1| \cong |T_2| \cong 1$, Eq. (20) reduces to

$$(21)\quad \ln\left|\frac{E_i}{E_o}\right| = -\frac{2\pi d}{\lambda}\,\frac{a^2}{\sqrt{a^2-\sin^2\theta_B}}\,\tan\frac{\delta_e+\delta_m}{2}\,.$$

The quantity $\left|\frac{E_i}{E_o}\right|^2$ can be considered as the ratio of the power received after transmission through the dielectric sheet to the power received when the sample is removed. Defining this ratio of powers by $\frac{1}{r^2}$, Eq. (21) yields

$$(22)\quad \tan\frac{\delta_e+\delta_m}{2} = \frac{\lambda}{2\pi d}\,\frac{\cos\theta_B}{k_m}\ln r\,.$$

In addition, for small losses

$$(23)\quad \tan\delta_e + \tan\delta_m \cong \tan(\delta_e+\delta_m) \cong 2\tan\frac{\delta_e+\delta_m}{2}\,.$$

It is not necessary to take into account multiple reflections in the determination of "r" because the reflection coefficient at Brewster's angle is very small and the multiple reflections are negligible.

In deriving Eqs. (22) and (23) certain approximations have been employed, amounting basically to the omission of terms of the order of $\tan^2(\delta_e+\delta_m)$ with respect to unity. A detailed and rather involved calculation shows that the error is less than 1% when $\tan(\delta_e+\delta_m)$ is less than 0.1.

*At Brewster's angle, θ_B, the reflection coefficient is nearly zero, and the transmission coefficients $|T_1|$ and $|T_2|$ are close to unity.

Separation of the Losses

The calculations in the preceding section provide an expression for the sum of the loss angles. An experimental procedure providing the difference of the loss angles would permit the calculation of $\tan\delta_e$ and $\tan\delta_m$ individually. One such procedure consists of measuring the ratio of the reflection coefficient for parallel polarization to the reflection coefficient for perpendicular polarization, the angle of incidence being equal to Brewster's angle for both cases. Unfortunately the determination of this ratio usually requires the measurement of an extremely high power ratio which is difficult to measure accurately with limited transmitted power. Methods utilizing phase measurements or a measurement of the standing wave ratio would permit separation of the losses, but they would require supplementary apparatus and would lead to other experimental difficulties.

Using Eqs. (5) and (9) it can be shown that the absolute value of the ratio of the reflection coefficient for parallel polarization to the reflection coefficient for perpendicular polarization (at the Brewster angle) is

$$(24)\quad \left|\frac{R_\parallel}{R_\perp}\right|_{\theta=\theta_B} = \frac{\frac{D}{S}\cot^2\theta_B - \frac{1}{2}\left(1+\frac{1}{k_e^2}\right)}{\sqrt{\left[\frac{D}{S}\cot^2\theta_B + \frac{1}{2}\left(3+\frac{1}{k_e^2}\right)\right]^2 + \frac{4(k_e k_m-1)^2}{k_e^2 S^2}}}$$

in which the notation has been simplified by defining:

$$(25)\quad D = k_e\tan\delta_e - k_m\tan\delta_m$$

$$(26) \qquad S = k_m(\tan \delta_e + \tan \delta_m) .$$

Terms in $\tan \delta_e$ and $\tan \delta_m$ of the second order and higher have been omitted in obtaining these relations.

The measured quantity is $\left|\frac{R_{\parallel}}{R_{\perp}}\right|^2$ which is defined as $\frac{1}{m^2}$. The quantity S can be be determined by the experimental procedures described above, so that the quantity D remains the only unknown.

As mentioned above, the measured ratio is generally very large, making "m" a very large number. Under these conditions Eq. (24) can be simplified, yielding

$$(27) \qquad D = \frac{2(k_e-k_m)}{m} + \frac{1}{2}\left(1+\frac{1}{k_e^2}\right) \cdot S \cdot \tan^2 \theta_B$$

Finally, combining equations (25) and (26) provides an expression for the dielectric losses alone:

$$(28) \qquad \tan \delta_e = \frac{S + D}{k_e + k_m} .$$

This relation permits the separation of the losses.

There is one source of error, neglected in deriving Eq. (27), which can modify appreciably the observed value of "m". This is an error which is due to multiple reflections in the sample. The necessary correction has been mentioned above in the discussion of the measurement of the reflection coefficient for parallel polarization. The calculation of this correction will now be considered.

The Effect of Multiple Reflections[5]

Assume that a plane electromagnetic wave strikes a dielectric sheet as indicated in Fig. 3. The problem loses no generality if the amplitude of the incident field is set equal to unity; also the time origin can be chosen to simplify the equations. The field at the origin is therefore set equal to $e^{j\omega t}$.

The wave reflected along the line AA' is given by

$$(29) \qquad e^{j\omega t} \cdot R_1 \cdot e^{-jK_o(x \sin \theta + y \cos \theta)}$$

where R_1 is the reflection coefficient of the upper surface of the dielectric, K_o is the propagation constant (or phase constant) in air, equal to $\omega\sqrt{\mu\varepsilon}$. The wave refracted along the line AD becomes

$$(30) \qquad e^{j\omega t} \cdot T_1 \cdot e^{-jK(x \sin \psi - y \cos \psi)}$$

where K is the propagation constant in the dielectric, and the quantity T_1 is the coefficient of transmission through the upper surface of the dielectric. Continuing this calculation for the waves BB', CC', etc., an infinite geometric series is obtained for the reflected field. By taking the sum of this series and substituting Eqs. (5), (6), (9) and (10), it can be shown that the true reflection coefficient is given by:

$$(31) \qquad |R_t| = |R_1| \cdot \left|\frac{1 - e^{-j2dK \cos \psi}}{1 - R_1^2 e^{-j2dK \cos \psi}}\right| .$$

The direct application of this expression leads to a result much too complicated to be useful in the calculation of $|R_t|$. The situation is complicated further because the experimental data necessary to determine several quantities in Eq. (31) must themselves be corrected for the difference between R_t and R_1. However, under the assumption of small losses the following approximate solution can be used.

The correction to be applied to experimental data taken at the Brewster angle for the case of perpendicular polarization will be calculated first. Assuming that the losses are small Eq. (15) yields:

$$(32) \qquad {R_1}^2 = \left[\frac{k_e - k_m}{k_e + k_m}\right]^2 = \frac{1}{{u_t}^2} .$$

In the following, u_t signifies the true value of "u", defined by Eq. (15). The apparent or measured value, which is called u_a, differs from u_t because of multiple reflections. After substitution of the relations for K and $\cos \psi$, Eq.(31) takes the form

$$(33) \qquad |R_1^2| = \frac{1}{u_a^2} = \frac{1}{u_t^2} \cdot \frac{1 - 2B \cos \beta + B^2}{1 - \frac{2B}{u_t^2} \cos \beta + \frac{B^2}{u_t^4}}$$

where

$$(34) \qquad \beta = \frac{4\pi d}{\lambda} \cdot k_e \cos \theta_B$$

$$(35)\quad B = \exp\left[-\frac{4\pi d}{\lambda} k_e \cos\theta_B \tan\frac{\delta_e+\delta_m}{2}\right].$$

The problem is to calculate u_t, knowing u_a. The direct solution of Eq. (33) for u_t will not suffice for the quantities B and β can be calculated only when k_e is known and the calculation of k_e(from Eqs. (12) and (15)) requires a knowledge of u_t. The simultaneous solution of Eqs. (12), (15) and (33) leads to a transcendental equation which must be solved by special methods.

If the value of k_m is near unity at the frequencies of interest, the following process furnishes a simple method for finding k_e and k_m. Assume temporarily that $k_m = 1$. Eq. (12) becomes

$$(36)\qquad \tan^2\theta_B = k_{e1}$$

where k_{e_1} is a first approximation to k_e. Using this value of ke_1 an approximate value of u_t can be calculated from Eq.(15).

$$(37)\qquad u_{t1} = \frac{k_{e1}+1}{k_{e1}-1}.$$

Substitution of Eq. (22) in Eq.(15) yields

$$(38)\qquad B = \exp\left[-2\frac{k_e}{k_m}\cos^2\theta_B \cdot \ln r\right].$$

If Eq. (33) is solved for cos β it is found that

$$(39)\qquad \cos\beta = \frac{u_a^2(1+B^2) - u_t^2\left(1+\frac{B^2}{u_t^4}\right)}{2B(u_a^2-1)}.$$

Eq. (38), the approximations (36) and (37), and experimental data for u_a, r, and θ_B yield an approximate value for cos β and thus for β. When this first value β_1, is substituted in Eq. (34) a second approximation, ke_2, is obtained. Solution of Eq. (12) yields

$$(40)\qquad k_m = \frac{k_e^2 + \tan\theta_B}{k_e(1+\tan^2\theta_B)},$$

and when ke_2 is substituted in this equation, a second approximation, km_2, is obtained for the relative permeability.

This process is repeated until the errors in the values of k_e and k_m are compatible with the precision of measurement, or until k_e and k_m no longer change appreciably from one approximation to the next. The process usually converges very rapidly and only one or two approximations are needed.

The ratio of reflection coefficients used in obtaining the separation of loss angles must also be corrected for multiple reflections. The quantity m^2 was defined by the ratio $\left|\frac{R_\perp}{R_\parallel}\right|^2$. This ratio will now be called m_t^2 and the measured ratio will be designated by m_a^2. Starting with Eq.(31) it can be shown that

$$(41)\qquad m_t^2 = m_a^2\left[1 - \frac{2B}{u_t^2}\cos\beta + \frac{B^2}{u_t^4}\right].$$

Since the quantities required for calculating u_t^2, β and B are determined by the above process, the correction for m_t^2 is easily obtained.

Summary of Measurement Procedure

The experimental application of the methods developed here requires the following measurements.

Using parallel polarization:

1. Brewster's angle θ_B;
2. The effective transmission coefficient, $\frac{1}{r}$, with $\theta = \theta_B$.
3. The reflection coefficient, $\frac{1}{u_a}$, with $\theta = \theta_B$.

Using perpendicular polarization:

4. The reflection coefficient, $|R_\perp|$, with $\theta = \theta_B$.

In addition:

5. The wavelength;

6. The thickness of the sample.

For many dielectrics it is known in advance that $k_m = 1$ and $\tan \delta_m = 0$; for these cases measurements 1 and 2 as well as 5 and 6 are sufficient. In 2 and 4 it is necessary to displace the receiving horn a quarter of a wave to compensate for errors due to standing waves. In 3 this displacement is usually not necessary because of the very small reflection coefficient.

Simultaneous solution of the above equations, using experimental data 1 to 6 enables the determination of the dielectric and magnetic properties of the sample. If multiple reflections in the sample are eliminated by using a wedge-shaped sample or by matching the rear surface of the sample to air, the calculations are straightforward and present no difficulty. When the sample is in the form of a sheet with parallel surfaces however, the measured quantities u_a and m_a must be corrected by means of the process described above.

<u>The Measurement Apparatus</u>

The experimental apparatus constructed for making the measurements consists essentially of three parts;

1. A transmitter consisting of a Klystron oscillator and its power supplies, a waveguide system with a wavemeter, a matching section, an attenuator, and an antenna;

2. A receiver, consisting of an antenna, a calibrated attenuator, a matching plunger, a crystal and a sensitive galvanometer;

3. Mechanical supports for the transmitter, the receiver and the dielectric sample; with provisions for accurately measuring angles, for displacing the receiving antenna a quarter wavelength, and for changing the plane of polarization.

The photographs in Fig. 4 show the apparatus in position for measuring transmission through the sample at Brewster's angle. Details of the transmitter and the receiver are also visible.

The transmitter and the receiver are mounted on movable arms with provisions for measuring angles. The sample is mounted with its surface perpendicular to a third movable arm which is able to pass under the other two arms to permit study of the case of normal incidence. The dielectric sheet can be removed easily from its holder for the measurement of direct transmission. The sheet between the sample and the receiver is an attenuator for reducing the error due to standing waves between the horns. A micrometer head can be seen at the base of the receiver mounting. This permits measurement of the quarter-wave displacement for compensating standing wave errors.

Fig. 4 is an enlarged view of the transmitter. The Klystron at the left is followed by an attenuator composed of a sheet of fiber (painted with India ink) which slips through a slot in the waveguide. The wavemeter is of the coaxial type. This type of construction was chosen because the calibration of a coaxial wavemeter does not depend on the diameter of the resonant cavity and this diameter consequently does not have to be held to close tolerances. The short-circuiting plunger of the wavemeter employs a double choke system to provide a good electrical short-circuit without requiring a perfect mechanical contact between the plunger and the coaxial conductors. The plunger is attached rigidly to a micrometer head and its displacement can be measured to a precision of 0.01 mm. The radii of the coaxial conductors were chosen so that the only mode capable of propagating is the fundamental mode of the coaxial guide. The matching section consists of an adjustable screw projecting through a slot in the guide. To match the guide to the antenna, the depth of the screw in the guide and its position along the axis of the guide are varied.

The transmitting and receiving antennas are identical and are constructed in the form of rectangular horns. Their radiation patterns are narrow enough (about 6.5 degrees to the half-power points) to avoid the effects of refraction at the edges of the dielectric sample. Each of the horns can be rotated about its axis to permit changing the polarization of the wave.

The receiving antenna is attached to a calibrated cut-off attenuator as indicated in the photograph (Fig. 4C) of the details of the receiver. Attenuation is determined by measuring the displacement of a dielectric rod which changes the length of the cut-off portion of the guide.

Fig. 5 shows a cross section of the attenuator. The important features of its construction are indicated on the drawing. Calibration of the attenuator in its linear region requires a knowledge of the precise diameter of the cut-off waveguide. The attenuator can be removed easily from the circuit, if desired, the detector then being coupled directly to the receiving horn. This is desirable when measuring Brewster's angle on a new dielectric, for the reflected power can be so small that the minimum attenuation of the attenuator could substantially reduce the sensitivity of the system.

The detector consists of a crystal, mounted across the waveguide. The rectified current is measured by a sensitive galvanometer. The crystal mount is composed of a short section of waveguide closed at the ends by two mica windows. The crystal and its contact wire are placed in the interior. Behind the crystal is a short-circuiting plunger (of the non-contacting, double choke type) to match the crystal to the waveguide.

Additional Considerations

There are several factors which can cause small errors or require modification of the experimental procedure. One of these is the fact that formulas derived for a plane wave have been employed in the calculations, while the wave used for the measurements approaches a spherical shape. The result obtained by assuming a plane wave is equivalent to the average of several experimental results at incidence near the Brewster angle. The error due to this effect has been discussed and examined experimentally by others[6] and it is concluded that the error is very small in practice. Another factor to be considered is the fact that the power density in a spherical wave is inversely proportional to distance. Because of this, a non-uniform amplitude distribution exists inside the sample. The theoretical analysis of this effect has not been made but it is evident that the use of a thin sample will reduce the error.

The similarity of the propagation constants of plane waves and spherical waves seems to indicate that the errors are small in most cases, although several of the formulas in which multiple reflections enter should be employed with caution.

Instability and drift of the transmitter frequency have only a small effect on the measurements, which are principally measurements depending on amplitudes and not on phase differences. However, the precision in the measurement of the tangent of the loss angles is directly proportional to the precision of frequency measurement. It is thus necessary to determine the wavelength rather carefully. On the other hand the determination of Brewster's angle (and consequently k_e) is not affected in the least by slight variations in frequency.

The presence of a slight radiation polarized at 90° to the principal field can cause rather serious errors in experiments requiring the measurement of very small reflected powers. In order to insure that this effect will be negligible the detector response should fall from maximum to a value of practically zero when the receiving horn is rotated 90° in its holder. For measurement of the sum of the losses this effect does not enter in the calculation of correcting terms, which in themselves are small.

In the determination of Brewster's angle there is a possibility of direct transmission of power between the two antennas. This power, even though small, can set up standing waves by interfering with the small reflected power. The use of a sheet of resistance paper or of rubber loaded with iron powder in the direct line between the antennas greatly reduces the error due to this effect. In addition the effect of multiple reflections in the sample must be considered. These two effects cause the curve of received power as a function of the angle of incidence to assume a "wave-like" appearance in the region of the minimum. Brewster's angle can be found quite accurately, however, by tracing the average of the experimental curve.

Experimental Results

Measurements were carried out on several dielectrics, in order to verify the theory and to examine the usefulness of the methods. Table I presents a partial summary of these results. All measurements were performed at a wavelength of 8.36 millimeters, corresponding to a frequency of 35,900 megacycles per second.

For most of the measurements the errors, estimated from the dispersion of experimental results, appear to be smaller than ± 6%; for glass and paraffin the measurements were dispersed by only ± 3%. The indicated values of k_m were determined as a check of the method, for it is known that $k_m = 1$ for these materials. In comparing the measured values with the values in the last two columns of the table, it should be remembered that these latter values were measured at a wavelength of 1.2 centimeters. In addition the materials may not have had exactly the same composition and consequently should not be expected to compare rigorously. This fact

Characteristics of Several Materials
at 8.36 mm Wavelength

Material	k_e	tan δ_e	k_m	Handbook[7] Values at 1.2 cm k_e	Handbook[7] Values at 1.2 cm tan δ_e
Bakelite	3.85	0.0526	0.966	3.5-4.5	0.009-0.052
Glass	7.24	0.0234	0.992	4.0-8.0	0.002-0.015
Polystyrene	2.52	0.0092	-	2.54	0.0012
Paraffin	2.46	-	-	2.25	0.0002

is evident in the case of polystyrene which exhibits a tan δ_e much larger than might be expected; the polystyrene is most certainly of inferior quality. The difference in the k_e of paraffin is probably due to the mode of preparation. In this case the value of tan δ_e is below the lower limit imposed by the sensitivity of the method; the error is so large that it is impossible to give a meaningful result.

The above calculations for non-magnetic dielectrics result in permeabilities very close to unity showing that the method is practical and that it leads to an acceptable precision. To examine the case of magnetic dielectrics, measurements were undertaken on a sheet of rubber loaded with iron powder. Although these measurements had not been completed at the time the author finished his studies at the University of Grenoble, it appeared that the method could provide satisfactory results.

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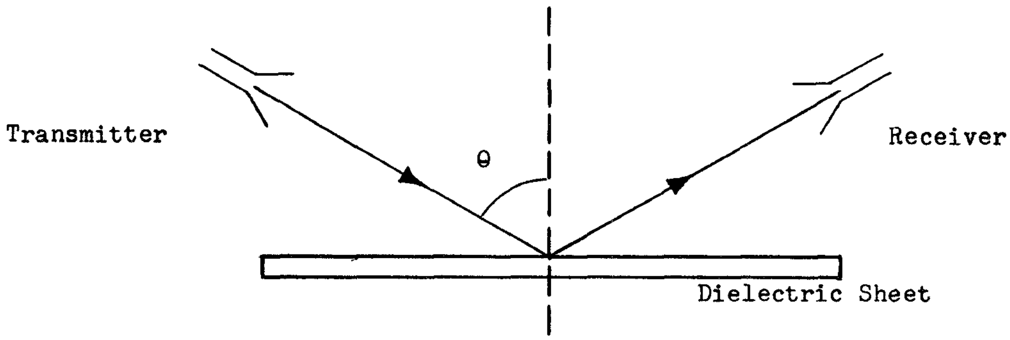

FIG. 1

Free space reflection from a dielectric sheet.

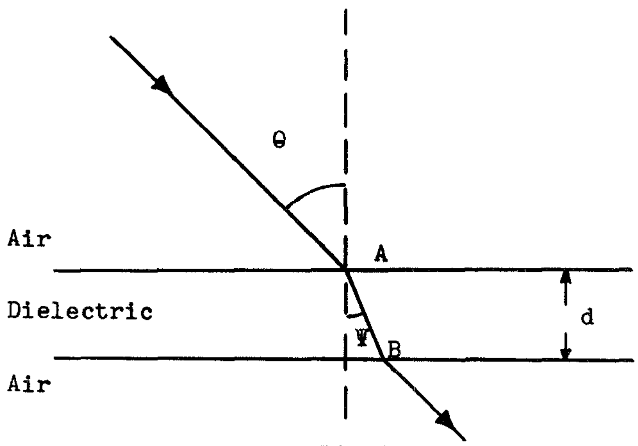

FIG. 2

Refraction at the boundary between two dielectrics.

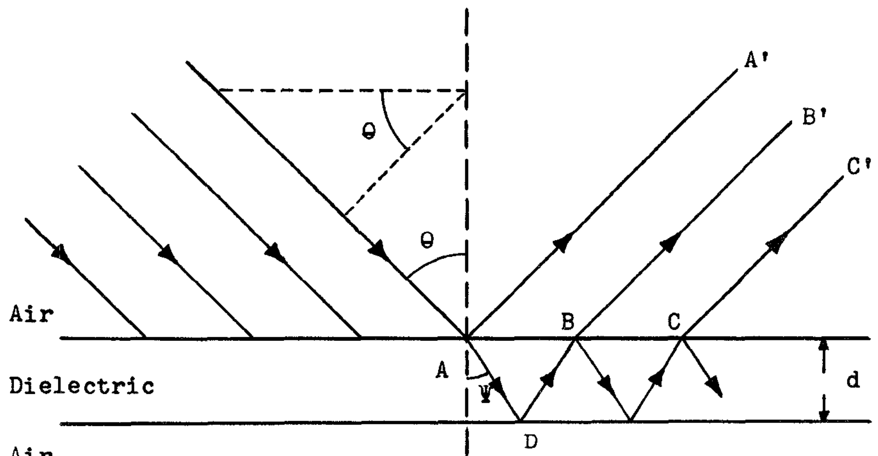

FIG. 3

Multiple reflections in a dielectric sheet.

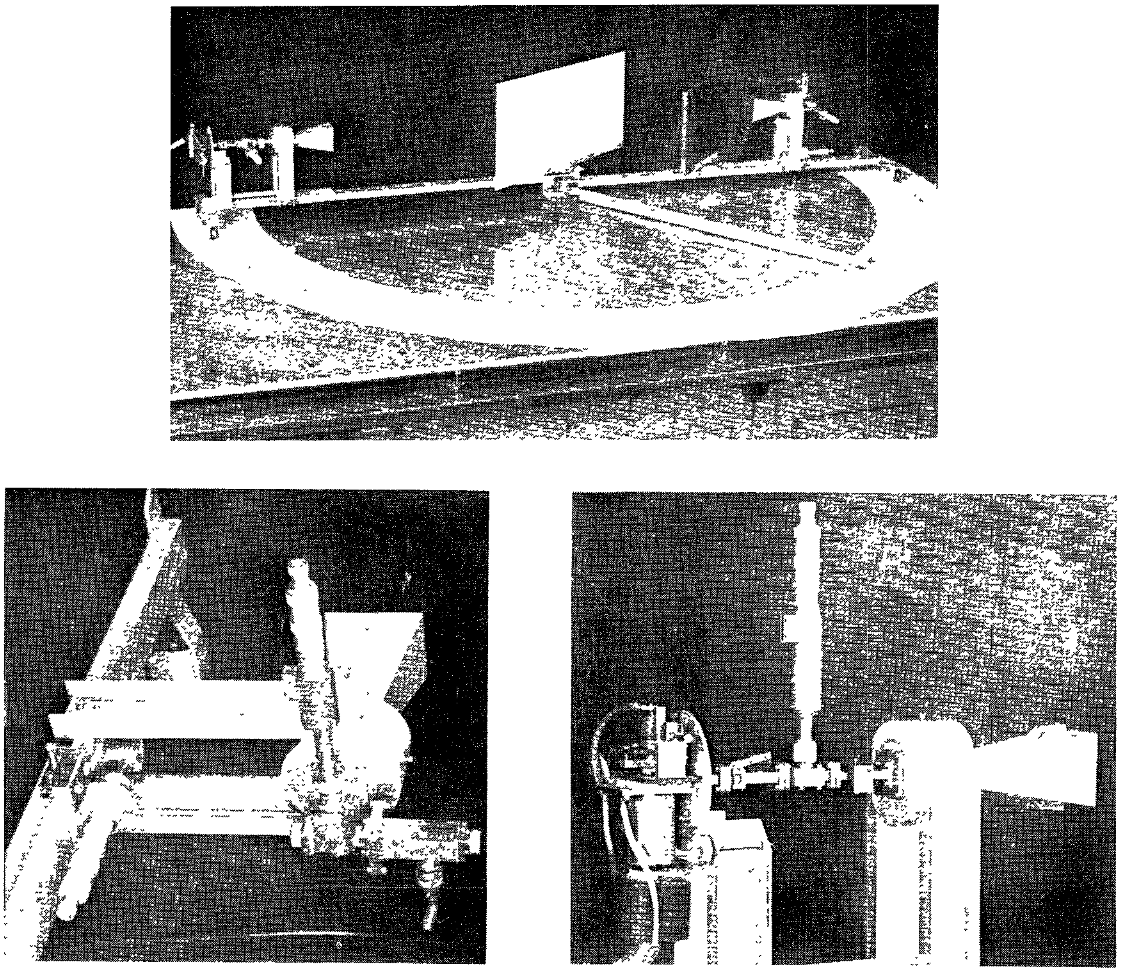

Fig. 4 - Measuring equipment: (a) general view; (b) the transmitter (c) the receiver.

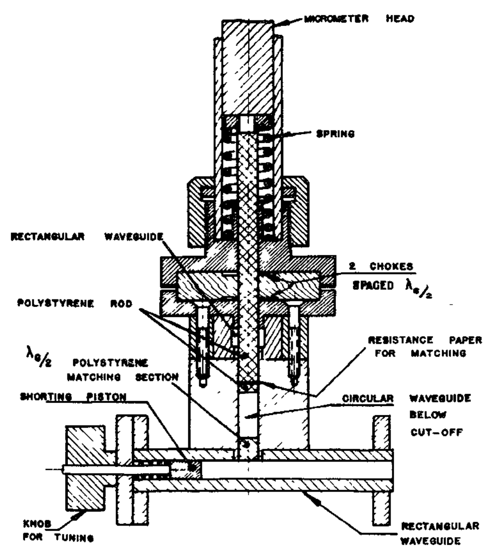

Fig. 5 - Cross sectional drawing of the cut-off attenuator.